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Integrated Flight/Propulsion Control: Subsystem Specifications for Performance

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In many systems such as integrated flight/propulsion control, it is desirable to implement the control law in a partitioned or decentralized architecture comprised of subsystems. For designs of this type, having a set of subsystem performance specifications early in the design cycle that is sufficient to guarantee overall performance of the final combined vehicle would be beneficial. A procedure is presented in this paper for deriving multiple subsystem specifications from a number of performance requirements on an integrated system. It is based on a structured singular-value analysis and generates specifications that are magnitude bounds on subsystem uncertainties. The specifications are derived from performance requirements that are provided in the form of bounds on transfer functions of the integrated system. This form allows the expression of model following, command tracking, robustness, and disturbance rejection requirements. The procedure is demonstrated on a short takeoff and landing aircraft design, in which engine specifications are derived from the aircraft's velocity and pitch rate tracking requirements.

I. Introduction

As discussed in Refs. 1 and 2, the exploitation of propulsion-generated forces and moments (via vectoring nozzles, ejectors, remote augmented lift systems, etc.) in the attitude control law of an aircraft can yield significant performance improvements in supermaneuverable air-to-air operation and in short takeoff and landing (STOL) operation. Further, using these devices introduces a dynamic coupling between the flight and propulsion systems that must be accounted for in the control system design. Control design procedures that specifically account for this coupling are referred to as integrated flight/propulsion control (IFPC) design procedures.

One of the issues in an IFPC approach is allowing for the fact that the overall system control, although integrated, will likely be decentralized. That is, the engine manufacturer will supply the propulsion control logic and the airframe manufacturer will supply the flight control logic. This decentralization is driven by the need to have the control designs performed by experts in the fields as well as the requirement for design accountability (e.g., the engine manufacturer must be responsible for the safety and performance of the engine). Required, therefore, is a procedure that will augment a normal set of subsystem specifications with additional requirements that, if met, will guarantee overall vehicle performance when the subsystems are combined into the integrated system.

Reference 2 introduced a technique for developing subsystem (e.g., propulsion) performance specifications that guarantee stability of the integrated system. This was based on small-amplitude analyses using linearized system models. The procedure hinged on the calculation of a scalar function of frequency referred to as l that bounds an uncertainty matrix representing an allowable deviation of the subsystems' performance away from a nominal (this nominal can be an assumed achievable performance or can be derived from a global design approach[3]). The bound l was then "partitioned" or distributed into a set of bounds for the individual subsystem transfer functions. The nominal performance combined with these tolerance bounds then represented a performance specification for each subsystem.

The goal of this paper is to describe a method for calculating l based on performance requirements in addition to stability requirements. This value of l will be referred to as l_{perf}, and l_{stab} will refer to the l from Ref. 2. When stated without a subscript in this paper, l can be replaced with either l_{perf} or l_{stab}. Once l_{perf} is calculated, the distribution of it among the subsystem transfer functions can proceed as described in Ref. 2.

The procedure is presented as the solution to a general problem. It is then applied to an IFPC problem in Sec. IV.

II. Problem Statement

Consider the block diagram shown in Fig. 1, where a system has been divided into a main system G and n subsystems T_j. The main system G consists of the mission-level plant and controller, such as an airplane and flight control system. A subsystem T_1 could, for example, be the propulsion system, consisting of the engine and its controller. If there are coupling paths between the subsystems, these are contained in the vector signals $\boldsymbol{u}_j$ and $\boldsymbol{u}_{cj}$, and the transfer function through G for those signals is unity.

A set of performance requirements are given that bound the transfer functions from $\boldsymbol{d}$ to $\boldsymbol{e}$. These requirements must remain satisfied even when the system is subject to variations in the subsystems T_j. Plant variations can also be taken into account, depending on whether these variations have known bounds, in which case they will be treated as additional performance requirements, or whether it is desired to produce specifications for these uncertainties by treating the plant as another subsystem.

The problem addressed in this paper is how to calculate specifications for the subsystems in this situation, given a main system design that includes the performance requirements and the nominal

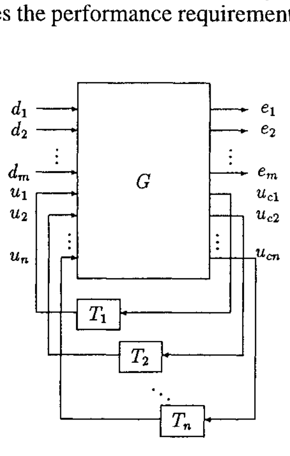

Fig. 1 Generic partitioned system showing subsystems.

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values for G and T_j. These specifications will bound the deviations from the nominal subsystem transfer functions and guarantee the performance of the integrated system.

III. Method of Analysis

The system of Fig. 1 can be put in the form of Fig. 2, where M is the known part of the system and Δ is unknown. The subsystems are described by the equation

$$T_j = Q_j + \Delta Q_j \tag{1}$$

where Q_j is the nominal value of T_j and ΔQ_j is the unknown deviation from the nominal. The interconnection matrix M is created by gathering the main system G and the nominal subsystem transfer functions Q_j together (demonstrated in Appendix B for the example in Sec. IV). The matrix Δ is made up of the subsystem uncertainties ΔQ_j:

$$\Delta = \begin{bmatrix} \Delta Q_1 & 0 & \cdots & 0 \\ 0 & \Delta Q_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta Q_n \end{bmatrix} \tag{2}$$

For this procedure, it is assumed that the performance requirements of the total system can be expressed in terms of bounds on the transfer functions from the inputs d_i to the outputs e_i. That is,

$$\left|M_{\Delta e_i d_i}(j\omega)\right| < b_{pi}(\omega) \tag{3}$$

Here M_Δ is the transfer matrix from $\boldsymbol{d}$ to $\boldsymbol{e}$ including the effects of Δ: $M_\Delta = M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21}$, and $M_{\Delta e_i d_i}$ is the transfer function in that matrix from d_i to e_i. The function b_{pi} is a scalar bound that can vary with frequency, and each transfer function $M_{\Delta e_i d_i}$ is SISO (single input, single output). In general, these transfer functions can be MIMO (multi-input, multi-output) with a scalar bound, but for simplicity of the notation, only SISO transfer functions are considered in this paper.

This form of the performance requirements can be used to describe disturbance rejection requirements when d_i is a disturbance input and e_i is the affected variable. Model-following or command tracking requirements can be used when $M_{\Delta e_i d_i} = (P_i)_{\text{actual}} - (P_i)_{\text{desired}}$. In this case, e_i would be the model-following or model-tracking error due to an input command d_i. Plant variations with known bounds can be treated as a performance requirement by defining b_{pi} to be the inverse of the bound on the plant uncertainty, and e_i is the input to the uncertainty and d_i is the output from the uncertainty.

Some weighting matrices must be defined to normalize the system as shown in Fig. 3 and facilitate the structured singular-value analysis. The normalized interconnection matrix will be called $\bar{M}$, and the normalized uncertainty matrix will be called $\bar{\Delta}$. The performance weighting matrices w_{p_i} are defined in terms of the performance bounds:

$$w_{p_i}(\omega) = b_{pi}^{-1}(\omega) \tag{4}$$

so that the performance requirements of Eq. (3) can be stated as bounds on the normalized transfer functions from the inputs d_i to the weighted outputs $\bar{e}_i$:

$$\left|\bar{M}_{\bar{\Delta}\bar{e}_i d_i}(j\omega)\right| < 1 \tag{5}$$

where $\bar{M}_{\bar{\Delta}} = \bar{M}_{11} + \bar{M}_{12}\bar{\Delta}(I - \bar{M}_{22}\bar{\Delta})^{-1}\bar{M}_{21}$.

The subsystem weighting matrices w_{s_j} are used in the following way. The singular value is used to measure the size of the weighted uncertainty matrix Δ_w,

$$\Delta_w = \begin{bmatrix} w_{s_1}^{-1}\Delta Q_1 & 0 & \cdots & 0 \\ 0 & w_{s_2}^{-1}\Delta Q_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{s_n}^{-1}\Delta Q_n \end{bmatrix} \tag{6}$$

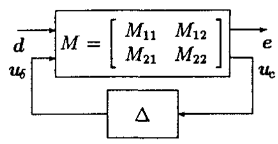

Fig. 2 Simplified system: M-Δ diagram.

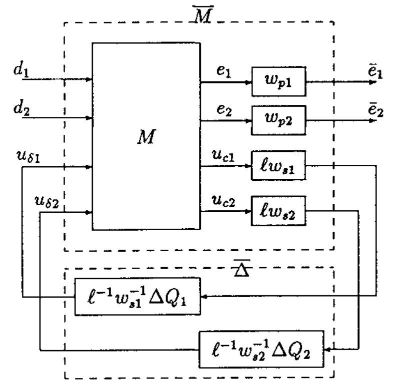

Fig. 3 Normalized system showing weighting matrices.

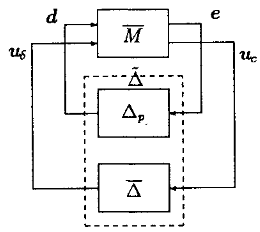

Fig. 4 M-Δ diagram showing performance block Δ_p.

and will be bounded by a scalar function of frequency called l:

$$\bar{\sigma}\{\Delta_w(j\omega)\} < l(\omega) \tag{7}$$

The normalized matrix $\bar{\Delta}$ is

$$\bar{\Delta} = l^{-1}\Delta_w \tag{8}$$

so that the subsystem uncertainty bound is equivalent to

$$\bar{\sigma}\{\bar{\Delta}\} < 1 \tag{9}$$

Since Δ_w is block diagonal [Eq. (6)] and using Eq. (7), the individual blocks are each separately bounded by l. So, the subsystem specifications become

$$\bar{\sigma}\left\{w_{s_j}^{-1}\Delta Q_j(j\omega)\right\} < l(\omega), \qquad \text{or} \qquad \bar{\sigma}\{\Delta Q_j(j\omega)\} < l(\omega)w_{s_j} \tag{10}$$

The subsystem weighting matrices w_{s_j} are chosen to distribute the available robustness among the subsystems. A larger value of w_{s_j} will provide more freedom to the jth subsystem. It will also reduce the freedom (tighten the specifications) on the other subsystems, because the value of l will be reduced (l depends on the weighting matrices). Usually the w_{s_j} are constant, but they can vary with frequency. For MIMO subsystems, the available robustness can be further distributed among each transfer function using the method described in Ref. 2.

The issue in generating subsystem specifications is determining how "big" the subsystem uncertainty matrices ΔQ_j can get without degrading the performance of the overall system beyond the bounds specified in Eq. (5). As defined in Eq. (10), l_{perf} is the limit on the size of the weighted ΔQ_j.

The value of l_{perf}, which limits the size of the subsystem uncertainty matrix, is determined from a structured singular-value (SSV) analysis as follows. Note that $\bar{M}$ and $\bar{\Delta}$ depend on l (see Fig. 3). Consider the system shown in Fig. 4. Doyle presents a theorem in Ref. 4 for robust performance that can be stated for this problem as follows.

Theorem 1. If $\mu_{\tilde{\Delta}}\{\bar{M}\} \leq 1$ and $\bar{\sigma}\{\bar{\Delta}\} < 1$, then $\bar{\sigma}\{\bar{M}_{\bar{\Delta}}\} < 1$, where the SSV μ is calculated with respect to the structure defined by

$$\tilde{\Delta} = \begin{bmatrix} \Delta_p & 0 \\ 0 & \bar{\Delta} \end{bmatrix} \tag{11}$$

In this matrix, Δ_p is a fictitious uncertainty, or performance block, that connects the output $\boldsymbol{e}$ to the input $\boldsymbol{d}$ (Fig. 4). This matrix is discussed in Refs. 4 and 5 and will be briefly described in the STOL aircraft example. It allows the performance requirements to be treated in parallel with the stability requirement.

The result of the theorem, that $\bar{\sigma}\{\bar{M}_{\bar{\Delta}}\} < 1$, will guarantee that the performance requirements of Eq. (5) are met, since

$$\left|\bar{M}_{\bar{\Delta}\bar{e}_i d_i}\right| \leq \bar{\sigma}\{\bar{M}_{\bar{\Delta}}\} \tag{12}$$

Hence the procedure to generate subsystem specifications reduces to satisfying the first two conditions of the theorem.

The first condition is satisfied by finding a function l_{perf} so that $\mu_{\tilde{\Delta}}\{\bar{M}\} \leq 1$.

The second condition is used to generate the subsystem specifications as shown in Eqs. (9) and (10).

Efficient methods for calculating l_{perf} are beyond the scope of this paper. Instead, a bisection algorithm was used for the calculation in the example below. In particular, initial upper and lower estimates of $l_{\text{perf}}(\omega)$ were made, $\mu_{\tilde{\Delta}}\{\bar{M}\}$ was calculated for the mean of these estimates, and then one estimate was set equal to the mean depending on whether $\mu_{\tilde{\Delta}}\{\bar{M}\} < 1$ or $\mu_{\tilde{\Delta}}\{\bar{M}\} > 1$. An iteration is needed because $\bar{M}$ depends on l_{perf}. (In the example below, the iteration is completed when the lower estimate converged to between 90 and 100% of the upper estimate.)

IV. Example

In this section, the SSV analysis is applied to a STOL aircraft. This aircraft is divided into three systems: 1) the main system (rigid-body motion and control system), 2) a propulsion subsystem with aft and ventral nozzles, and 3) an aerodynamic control subsystem with three control surfaces. The performance measures for this example are based on how well the aircraft tracks airspeed commands and pitch rate commands as defined in Ref. 6, which were based in part on the military specifications.[7] The objective of this example is to demonstrate how specifications can be generated using these airspeed and pitch rate tracking requirements. The specifications will define how well the propulsion and aerodynamic control subsystems must track their commands.

A. STOL Aircraft Equations

The linear model for this aircraft is the same one used in Ref. 2, which was generated at an altitude of 100 ft, a Mach number of 0.18, and a flight path angle of 3 deg using a mass of 839 slugs.

A block diagram of the aircraft and control systems is shown in Fig. 5. The plant is described by the blocks P and C_a, with the equations

$$\dot{x} = A_p x + B_p \boldsymbol{u} \tag{13}$$

$$y = C_a \boldsymbol{x} \tag{14}$$

The values of the state matrices or transfer functions shown in Fig. 5 are given in Appendix A. The aircraft states are

$$x = \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} = \begin{bmatrix} \text{forward velocity, ft/s} \\ \text{vertical velocity, ft/s} \\ \text{pitch rate, deg/s} \\ \text{pitch angle, deg} \end{bmatrix} \tag{15}$$

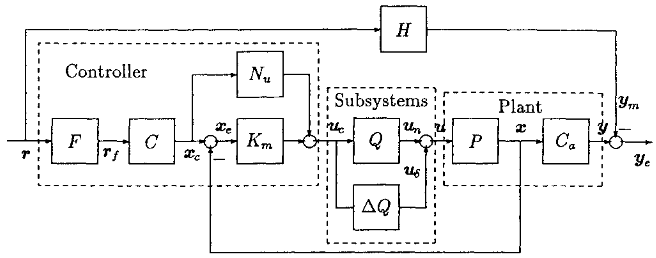

Fig. 5 Block diagram of STOL aircraft.

The rigid-body motion of the aircraft defines the plant P, which responds to inputs $\boldsymbol{u} = [F_a \quad F_v \quad \delta H \quad \delta N \quad \delta F]'$. Because of the performance requirements, the outputs of interest are the airspeed and pitch rate, $\boldsymbol{y} = [V \quad q]'$.

There are many options available for designing the flight controller, and the specification procedure does not depend on the method chosen. For this example, the flight control system consists of the blocks F, C, K_m, and N_u. It takes the reference inputs $\boldsymbol{r} = [V_{\text{sel}} \quad \delta_{\text{st}}]'$ and generates commands $\boldsymbol{u}_c$ to the subsystems. Here, V_{sel} is the airspeed command and δ_{st} is the stick input for the pitch rate command. The equations are

$$\boldsymbol{x}_c = CF\boldsymbol{r} \tag{16}$$

$$\boldsymbol{u}_c = K_m(\boldsymbol{x}_c - \boldsymbol{x}) + N_u \boldsymbol{x}_c \tag{17}$$

The reference inputs are shaped by the filter F and processed through C to generate a consistent set of state commands $\boldsymbol{x}_c$. The control commands $\boldsymbol{u}_c$ are produced by the feedback and feedforward matrices K_m and N_u.

The subsystems consist of the blocks Q and ΔQ, with the equations

$$\dot{\boldsymbol{u}}_n = A_q \boldsymbol{u}_n + B_q \boldsymbol{u}_c \tag{18}$$

$$\boldsymbol{u}_\delta = \Delta Q\, \boldsymbol{u}_c \tag{19}$$

$$\boldsymbol{u} = \boldsymbol{u}_n + \boldsymbol{u}_\delta \tag{20}$$

The propulsion subsystem is responsible for producing u_1 and u_2, which are an axial thrust F_a acting through the center of gravity and a vertical thrust F_v acting approximately 10 ft behind the center of gravity.

The aerodynamic control surfaces are the horizontal tail and leading-edge and trailing-edge flaps, with deflections $[u_3 \quad u_4 \quad u_5] = [\delta H \quad \delta N \quad \delta F]'$.

These two subsystems are grouped in $T = Q + \Delta Q$. Recall that the nominal subsystem transfer functions are in Q, and the discrepancy between the nominal and what may actually be manufactured is handled by the additive uncertainty matrix ΔQ. Consequently, the plant P actually responds to a combination of the nominal command $\boldsymbol{u}_n$ and the uncertain command $\boldsymbol{u}_\delta$.

The H block represents the desired performance of the system. It is the desired response of the outputs $\boldsymbol{y}$ to the reference inputs $\boldsymbol{r}$, and it produces the model outputs $\boldsymbol{y}_m$. The difference between the actual outputs and the model outputs is $\boldsymbol{y}_e$, which consists of the airspeed tracking error V_{error} and the pitch rate tracking error q_{error}. The matrix M_Δ is the whole system shown in Fig. 5, with inputs $\boldsymbol{r}$ and outputs $\boldsymbol{y}_e$.

B. Main System Design

Comparing Figs. 1 and 5, the main system G consists of the "controller" (F, C, K_m, and N_u), the "plant" (P and C_a), the nominal subsystem performance Q, and the desired performance H. Designing the main system therefore refers to the design of the matrices that make up the controller and the nominal subsystem performance.

The design of the main system can be accomplished in a variety of ways. The choice does not affect the specification generation procedure presented in this paper. For example, one technique (that was not used here) would be to use the global design approach offered by Garg et al.[6] and then partition their control into a main system controller and subsystem controllers, as shown in Ref. 3.

For this example, a simpler method was used to obtain the main system G, and the nominal matrix Q was simply chosen based on reasonable bandwidths for the subsystems.

The objective of the main-system design is to obtain the desired responses to the reference commands. The desired responses were taken from Ref. 6:

$$\frac{V}{V_{\text{sel}}} = \frac{0.04(s+3.13)}{s^2+0.6408s+0.1296} \tag{21}$$

$$\frac{q}{\delta_{\text{st}}} = \frac{35.12(s+0.5)}{s^2+3.9872s+5.0176} \tag{22}$$

To accomplish this objective, K_m, N_u, C, and F were designed as follows. A simple linear quadratic regulator (LQR) design produced the state feedback matrix K_m. The feedforward matrix N_u was obtained by determining the steady-state control necessary to cause the steady-state values of $\boldsymbol{x}$ to match a step input command $\boldsymbol{x}_c$. The command input logic C provides a consistent set of state commands by determining proper values of u and w for the selected airspeed V_{sel} and by integrating the pitch rate command δ_{st} to get the pitch command θ_c (see Appendix A for the equations). The filters F, which shape the inputs V_{sel} and δ_{st}, were designed by iteration using a visual comparison of the frequency-domain responses at each step. The resulting responses of the nominal system (with $\Delta Q = 0$) are within 16% of the desired responses at each frequency.

C. Performance Requirements

A complete set of performance requirements consists of a specification of the nominal performance and a bound on the deviation from the nominal performance. The nominal performance is given in Eqs. (21) and (22). What remains is to decide how well the performance must be maintained when the system is subjected to the uncertainties in the subsystems.

For the STOL aircraft example, the allowed deviation of the aircraft's airspeed and pitch rate responses from the nominal responses must be defined; i.e., b_{pi} in the following equation must be defined:

$$|(P_i)_{\text{actual}}(j\omega) - (P_i)_{\text{nominal}}(j\omega)| < b_{pi}(\omega) \tag{23}$$

where $(P_1)_{\text{actual}}$ is the transfer function from V_{sel} to V, $(P_2)_{\text{actual}}$ is the transfer function from δ_{st} to q, and $(P_i)_{\text{nominal}}$ is the desired transfer function.

The bound b_{p1} is shown in Fig. 6 as an allowed percentage deviation from the nominal velocity tracking transfer function. The nominal transfer function is the dotted line in Fig. 7, and the solid lines are upper and lower bounds defined by b_{p1}. The bound for pitch rate tracking is similar.

The performance requirements can now be stated as

$$\left|\frac{y_{e1}}{r_1}(j\omega)\right| = \left|\frac{V_{\text{error}}}{V_{\text{sel}}}(j\omega)\right| < b_{p1}(\omega) \tag{24}$$

$$\left|\frac{y_{e2}}{r_2}(j\omega)\right| = \left|\frac{q_{\text{error}}}{\delta_{\text{st}}}(j\omega)\right| < b_{p2}(\omega) \tag{25}$$

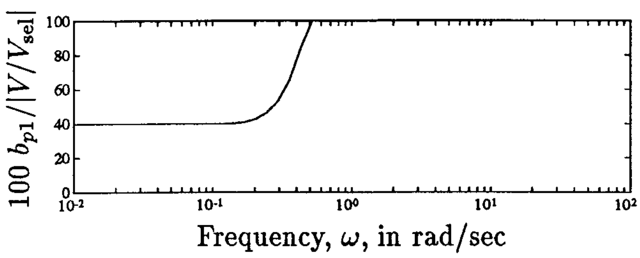

Fig. 6 Percent deviation bound for airspeed tracking performance.

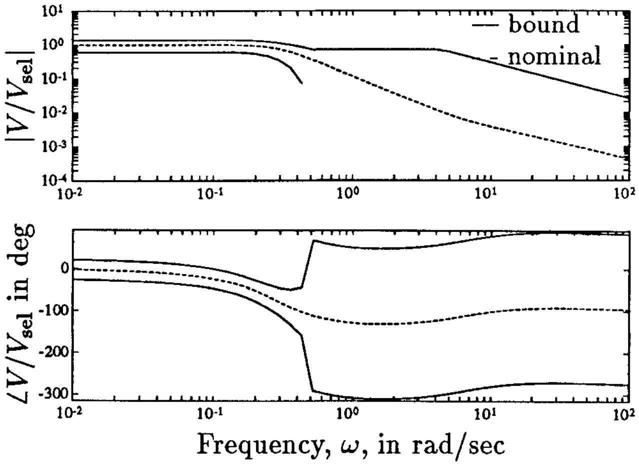

Fig. 7 Performance bounds for airspeed tracking.

These transfer functions are the diagonal entries in the matrix M_Δ:

$$M_\Delta = \begin{bmatrix} (P_1)_{\text{actual}} - (P_1)_{\text{nom}} & (P_{12})_{\text{actual}} - (P_{12})_{\text{nom}} \\ (P_{21})_{\text{actual}} - (P_{21})_{\text{nom}} & (P_2)_{\text{actual}} - (P_2)_{\text{nom}} \end{bmatrix} \tag{26}$$

where

$$(P_1)_{\text{actual}} - (P_1)_{\text{nom}} = \frac{y_{e1}}{r_1} \tag{27}$$

$$(P_{12})_{\text{actual}} - (P_{12})_{\text{nom}} = \frac{y_{e1}}{r_2} \tag{28}$$

$$(P_{21})_{\text{actual}} - (P_{21})_{\text{nom}} = \frac{y_{e2}}{r_1} \tag{29}$$

$$(P_2)_{\text{actual}} - (P_2)_{\text{nom}} = \frac{y_{e2}}{r_2} \tag{30}$$

The nominal transfer functions are those transfer functions from $\boldsymbol{r}$ to $\boldsymbol{y}$ with $\Delta Q = 0$ and are used to define the H block:

$$H = \begin{bmatrix} (P_1)_{\text{nom}} & (P_{12})_{\text{nom}} \\ (P_{21})_{\text{nom}} & (P_2)_{\text{nom}} \end{bmatrix} \tag{31}$$

Next, the structure used for the fictitious uncertainty Δ_p in the $\tilde{\Delta}$ matrix of Eq. (11) must be specified. This structure is dictated by the performance requirements. As explained by Doyle et al.,[5] the fictitious uncertainty satisfies

$$\bar{\sigma}\{\Delta_p(j\omega)\} < 1 \tag{32}$$

and it is attached to $\bar{M}$ in parallel with the actual uncertainty $\bar{\Delta}$, as shown in Fig. 4. This converts a performance robustness problem into a stability robustness problem as follows.

Since the two performance requirements for this example apply only to the diagonal elements of M_Δ, i.e., y_{e1}/r_1 and y_{e2}/r_2, the structure of Δ_p can be diagonal. Because of Eq. (32), the two entries of Δ_p satisfy

$$|\Delta_{pi}(j\omega)| < 1, \qquad i = 1, 2 \tag{33}$$

To guarantee stability of the augmented system of Fig. 4 using the small-gain theorem, the weighted transfer functions that occur in series with Δ_{pi} must satisfy

$$\left|\frac{y_{e1}}{r_1}(j\omega)\, b_{p1}^{-1}(\omega)\right| < 1 \tag{34}$$

$$\left|\frac{y_{e2}}{r_2}(j\omega)\, b_{p2}^{-1}(\omega)\right| < 1 \tag{35}$$

These inequalities are the performance requirements stated in Eqs. (24) and (25) but in the form of Eq. (5).

Incidentally, the nondiagonal terms of Eq. (26) are also bounded because there is a third loop in the $\bar{M} - \Delta_p$ feedback:

$$\left| \frac{y_{e1}}{r_2}(j\omega)\, b_{p1}^{-1}(\omega)\, \frac{y_{e2}}{r_1}(j\omega)\, b_{p2}^{-1}(\omega) \right| < 1 \tag{36}$$

D. Subsystems

A more detailed discussion of the propulsion and aerodynamic control subsystems can be found in Ref. 2. Briefly, the nominal subsystem transfer functions are

$$Q(s) = \mathrm{diag}\{q_1(s), q_1(s), q_2(s), q_2(s), q_2(s)\} \tag{37}$$

$$q_1(s) = \frac{10}{s+10} \tag{38}$$

$$q_2(s) = \frac{15}{s+15} \tag{39}$$

which means that the propulsion system is assumed to operate with a nominal bandwidth of 10 rad/s and the aerodynamic controls at 15 rad/s. These are assumed to be representative of reasonable goals for future aircraft systems.

The unknown part of the subsystem transfer functions are grouped in

$$\Delta Q = \begin{bmatrix} \Delta Q_{\mathrm{prop}} & 0 \\ 0 & \Delta Q_{\mathrm{aero}} \end{bmatrix} \tag{40}$$

with

$$\Delta Q_{\mathrm{prop}}(s) = \begin{bmatrix} \delta_1(s) & \delta_2(s) \\ \delta_3(s) & \delta_4(s) \end{bmatrix} \tag{41}$$

$$\Delta Q_{\mathrm{aero}}(s) = \begin{bmatrix} \delta_5(s) & 0 & 0 \\ 0 & \delta_6(s) & 0 \\ 0 & 0 & \delta_7(s) \end{bmatrix} \tag{42}$$

The interpretation of Eqs. (41) and (42) is that the propulsion system has a coupled response, with the transfer matrix

$$T_1(s) = \begin{bmatrix} \dfrac{F_a}{F_{ac}}(s) & \dfrac{F_a}{F_{vc}}(s) \\ \dfrac{F_v}{F_{ac}}(s) & \dfrac{F_v}{F_{vc}}(s) \end{bmatrix} = \begin{bmatrix} q_1(s)+\delta_1(s) & \delta_2(s) \\ \delta_3(s) & q_1(s)+\delta_4(s) \end{bmatrix} \tag{43}$$

and that the aerodynamic subsystems are independent of one another, where each control surface responds with the transfer function

$$\left(\frac{u_j}{u_{jc}}\right)(s) = q_2(s) + \delta_{j+2}(s), \qquad j = 3, 4, 5 \tag{44}$$

and

$$T_2(s) = \mathrm{diag}\left\{\left(\frac{u_j}{u_{jc}}\right)(s)\right\}, \qquad j = 3, 4, 5 \tag{45}$$

The subsystem specifications will be bounds on the δ_k, telling the subsystem designer how close T_j must match Q_j.

E. Calculating Subsystem Specifications

Before applying the SSV theorem for robust performance, the interconnection matrix M must be constructed and appropriate weighting matrices defined. The four submatrices of M shown in Fig. 2 can be calculated from the block diagram of Fig. 5, as described in Appendix B.

The resulting interconnection matrix is

$$M = \begin{bmatrix} 0 & C_a P(I + QK_mP)^{-1} \\ (I + K_mPQ)^{-1}(N_u + K_m)CF & -(I + K_mPQ)^{-1}K_mP \end{bmatrix} \tag{46}$$

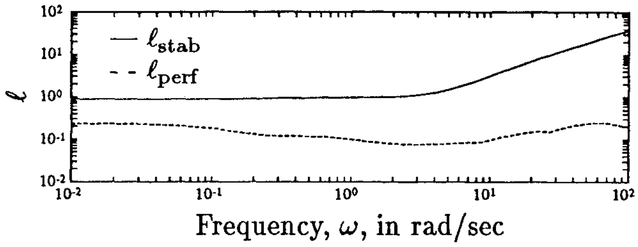

Fig. 8 Stability and performance bounds $l(\omega)$.

To normalize M (see Fig. 3), the output weighting matrices are defined as

$$w_{p1} = b_{p1}^{-1} \tag{47}$$

$$w_{p2} = b_{p2}^{-1} \tag{48}$$

$$w_{s1} = 1 \tag{49}$$

$$w_{s2} = 1 \tag{50}$$

where b_{pi} are defined in Eq. (23).

In this example, the weighting matrices for the subsystems are set to 1 to distribute the available robustness equally. If, however, the resulting subsystem specifications are too tight for the propulsion designer, for example, then the output weighting to the propulsion subsystem could be increased, resulting in relaxed specifications for the propulsion subsystem (and tightened aerodynamic subsystem specifications). If the specifications are too tight for all subsystems, then assuming that the performance requirements are reasonable, two possibilities exist. First, the main system design is not robust enough. This situation would be handled by a redesign. Second, the nominal transfer functions in Q are not accurate models of the subsystems, i.e., Q_j is not very near the actual value T_j. In that case, the subsystem models should be re-evaluated.

Applying the SSV theorem for robust performance, the bounding function l_{perf} was calculated using the bisection algorithm discussed earlier. The result is plotted in Fig. 8. Also shown for comparison is the l_{stab} from Ref. 2, which was calculated for stability requirements only.

Note that the function l is the main result. It is a bound on the maximum singular value of the weighted subsystem uncertainty matrix:

$$\bar{\sigma}\{\Delta_w(j\omega)\} < l(\omega) \tag{51}$$

Given l, individual bounds on each subsystem's performance can be obtained using the techniques provided in Ref. 2. The procedure depends upon a user-specified matrix $\overline{\Delta Q}$ that defines the distribution of available robustness (i.e., it is a degree of freedom available to the designer in the procedure). The result is

$$(\Delta Q_{\mathrm{prop}})_{ij}(j\omega) < \frac{(\overline{\Delta Q}_{\mathrm{prop}})_{ij}}{\bar{\sigma}\{\overline{\Delta Q}_{\mathrm{prop}}\}} l(\omega) \tag{52}$$

$$(\Delta Q_{\mathrm{aero}})_{ij}(j\omega) < \frac{(\overline{\Delta Q}_{\mathrm{aero}})_{ij}}{\bar{\sigma}\{\overline{\Delta Q}_{\mathrm{aero}}\}} l(\omega) \tag{53}$$

where for this example

$$\overline{\Delta Q}_{\mathrm{prop}} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{54}$$

$$\overline{\Delta Q}_{\mathrm{aero}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{55}$$

This reduces to the following specifications [see Eqs. (43) and (44)]:

Propulsion system:

$$|\delta_k| < b_{s1} = \tfrac{1}{2} l, \qquad k = 1, 2, 3, 4 \tag{56}$$

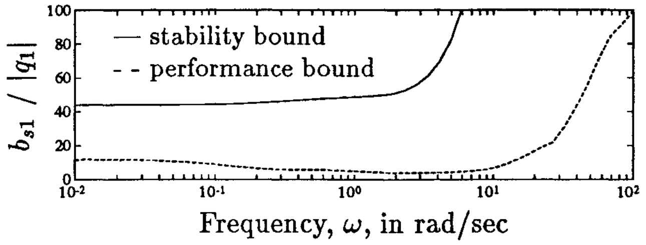

Fig. 9 Tracking specifications for propulsion subsystem.

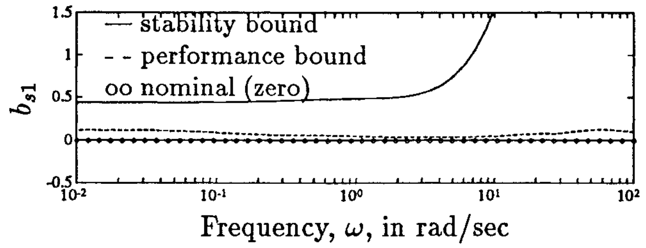

Fig. 10 Disturbance rejection specifications for propulsion subsystem.

Aerodynamic system:

$$|\delta_k| < b_{s2} = l, \qquad k = 5, 6, 7 \tag{57}$$

Equations (56) and (57) are the final result.

The bounds b_{sj} on δ_k can be supplied to the subsystem designer as the subsystem specifications. This version of the specifications is shown in Figs. 9 and 10. The solid lines are bounds based on l_{stab} and the dashed lines are based on l_{perf}. In Fig. 9, the bound on δ_k for $k = 1, 4$ is plotted as a percentage of the nominal subsystem transfer function. In Fig. 10, the bound on δ_k for $k = 2, 3$ is plotted directly (because the nominal disturbance rejection is zero). The bounds for the aerodynamic control surface tracking specifications (δ_k for $k = 5, 6, 7$) are two times less restrictive as the propulsion specification in Fig. 9. If the actual subsystem transfer functions T_j are designed with deviations δ_k below the dashed lines, then the aircraft will meet the performance requirements of airspeed and pitch rate tracking discussed in Sec. IV.C.

An obvious and reassuring feature of the results is that the subsystem specifications for performance are tighter than the specifications for stability. This makes sense because the performance requirements are in addition to the stability requirements, and the SSV theorem for robust performance includes robust stability.

V. Conclusions

A procedure has been developed that generates specifications for control design of subsystems in a partitioned system, such as within the context of IFPC. If the subsystem designs meet these specifications, then the performance of the total integrated system will be maintained.

This procedure has been demonstrated using a STOL aircraft model, and specifications on the propulsion subsystem and the aerodynamic controls were generated. Each specification is in the form of a bound on the allowed deviation of the subsystem from its nominal transfer function. If the subsystems are manufactured to meet these specifications, then the performance requirements of the aircraft with respect to tracking airspeed and pitch rate commands will be satisfied.

Appendix A: Equations of Motion

The STOL aircraft rigid-body dynamic equations that define the plant transfer matrix P are

$$\dot{\boldsymbol{x}} = A_p \boldsymbol{x} + B_p \boldsymbol{u}$$

$$\boldsymbol{y} = C_a \boldsymbol{x}$$

where

$$A_p = \begin{bmatrix} -5.893 \times 10^{-2} & 0.1068 & -0.6737 & -0.5557 \\ -0.2659 & -0.2665 & 3.400 & -8.027 \times 10^{-2} \\ -8.832 \times 10^{-2} & 0.4473 & -0.1949 & -4.818 \times 10^{-4} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0.1136 & 3.468 \times 10^{-2} & -4.960 \times 10^{-2} & 1 & 0 \\ -0.2321 & 6.925 \times 10^{-2} & -0.1455 & 0 & -1 \\ -1.310 & -0.4641 & 4.087 \times 10^{-2} & 0 & -4.125 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$C_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The nominal subsystem transfer matrix Q is defined by

$$\dot{\boldsymbol{u}}_n = A_q \boldsymbol{u}_n + B_q \boldsymbol{u}_c$$

where

$$A_q = \begin{bmatrix} -10 & 0 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & -15 & 0 \\ 0 & 0 & 0 & 0 & -15 \end{bmatrix}$$

$$B_q = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 & 15 \end{bmatrix}$$

The main control system is

$$\boldsymbol{u}_c = K_m \boldsymbol{x}_e + N_u \boldsymbol{x}_c$$

$$\boldsymbol{x}_e = \boldsymbol{x}_c - \boldsymbol{x}$$

where

$$K_m = \begin{bmatrix} 0.1594 & -0.1826 & -0.7579 & -0.4739 \\ 5.956 \times 10^{-2} & -2.314 \times 10^{-2} & -0.2538 & -0.2537 \\ -3.655 \times 10^{-2} & -3.828 \times 10^{-2} & 1.302 \times 10^{-2} & 0.1032 \\ 0.1378 & 1.015 \times 10^{-2} & -2.027 \times 10^{-2} & -9.007 \times 10^{-3} \\ 7.349 \times 10^{-2} & -0.1659 & -0.5957 & -0.3317 \end{bmatrix}$$

$$N_u = \begin{bmatrix} 0.2732 & 0.4369 & -3.819 & 0.1321 \\ 0.6949 & 1.069 & -9.837 & 0.2108 \\ -0.6102 & -0.9227 & 8.577 & -0.1965 \\ -2.646 \times 10^{-2} & -0.2392 & 1.874 & 0.5236 \\ -0.1929 & -0.1597 & 2.357 & -6.773 \times 10^{-2} \end{bmatrix}$$

The control logic C is

$$\dot{\theta}_c = q_c$$

$$q_c = r_2$$

$$u_c = V_{\text{sel}}$$

$$w_c = 0$$

It has a state-space description of

$$\dot{\theta}_c = A_c\theta_c + B_c\boldsymbol{r}_f$$

$$\boldsymbol{x}_c = C_c\theta_c + D_c\boldsymbol{r}_f$$

where

$$A_c = 0$$

$$B_c = [0 \quad 0.0611]$$

$$C_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$D_c = \begin{bmatrix} 0.9660 & 0 \\ 0 & 0 \\ 0 & 0.0611 \\ 0 & 0 \end{bmatrix}$$

The prefilters in F are

$$\frac{r_{f1}}{r_1} = \frac{(s+5.3)^2}{(s+0.45)(s+1000)}$$

$$\frac{r_{f2}}{r_2} = \frac{(s+0.5)(s+8.5)^2}{(s+2)^2(s+1000)}$$

and so

$$F = \operatorname{diag}\left\{\frac{r_{f1}}{r_1}, \frac{r_{f2}}{r_2}\right\}$$

Appendix B: Interconnection Matrix

The four submatrices of the interconnection matrix M are calculated as follows. The equation for the aircraft output is

$$\boldsymbol{y} = C_a P Q (I + K_m P Q)^{-1}(N_u + K_m) C F \boldsymbol{r} + C_a P (I + Q K_m P)^{-1} \boldsymbol{u}_\delta \tag{B1}$$

and the equation for the commanded control input is

$$\boldsymbol{u}_c = (I + K_m P Q)^{-1}(N_u + K_m) C F \boldsymbol{r} - (I + K_m P Q)^{-1} K_m P \boldsymbol{u}_\delta \tag{B2}$$

The equation for the desired response $\boldsymbol{y}_m$ is the same as $\boldsymbol{y}$ but with $\Delta Q = 0$, which means $\boldsymbol{u}_\delta = 0$, so

$$\boldsymbol{y}_m = C_a P Q (I + K_m P Q)^{-1}(N_u + K_m) C F \boldsymbol{r} \tag{B3}$$

Therefore

$$\boldsymbol{y}_e = \boldsymbol{y} - \boldsymbol{y}_m = C_a P (I + Q K_m P)^{-1} \boldsymbol{u}_\delta \tag{B4}$$

The matrix M is constructed by identifying the appropriate terms in the above equations for the following equation:

$$\begin{bmatrix} y_e \\ u_c \end{bmatrix} = M \begin{bmatrix} r \\ u_\delta \end{bmatrix} \tag{B5}$$

The resulting interconnection matrix is

$$M = \begin{bmatrix} 0 & C_a P (I + Q K_m P)^{-1} \\ (I + K_m P Q)^{-1}(N_u + K_m) C F & -(I + K_m P Q)^{-1} K_m P \end{bmatrix} \tag{B6}$$

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